

# Binary Evolution Leads to Two Populations of White Dwarf Companions

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## ABSTRACT

Planets and other low-mass binary companions to stars face a variety of potential fates as their host stars move off the main sequence and grow to subgiants and giants. Stellar mass loss tends to make orbits expand, and tidal torques tend to make orbits shrink, sometimes to the point that a companion is directly engulfed by its primary. Furthermore, once engulfed, the ensuing common envelope (CE) phase can result in the companion becoming fully incorporated in the primary's envelope; or, if the companion is massive enough, it can transfer enough energy to eject the envelope and remain parked in a tight orbit around the white dwarf core. Therefore, ordinary binary evolution ought to lead to two predominant populations of planets around white dwarfs: those that have been through a CE phase and are in short-period orbits, and those that have entirely avoided the CE and are in long-period orbits.

## 1. Introduction

Many intermediate-mass stars have either stellar companions or brown dwarf or planetary companions (Duquennoy & Mayor 1991; Raghavan et al. 2010; Schneider et al. 2011; Wright et al. 2011). These systems can sometimes remain dynamically quiet for hundreds of millions or billions of years, and then undergo relatively rapid changes when the primary star evolves off the main sequence.

Various recent works have investigated the post-main-sequence fates of two-body systems (Villaver & Livio 2007; Carlberg et al. 2009; Villaver & Livio 2009; Nordhaus et al. 2010) and of many-body systems (Veras et al. 2011; Veras & Tout 2012; Kratter & Perets 2012). The present work is generally concerned with the former type of system — i.e., the evolution of a single star and a single low-mass companion.

The status of planetary systems around post-main-sequence stars is starting to come into focus. Zuckerman et al. (2010) find evidence of heavy-element atmospheric pollution in  $\sim 1/3$  of DB white dwarfs, which they interpret as due to accretion of tidally shredded asteroids that were presumably scattered onto high-eccentricity orbits by distant planets. Maxted et al. (2006) found evidence of perhaps the best-characterized substellar companion to a white dwarf, via high-resolution spectroscopy that revealed radial velocity variations with period 2 hours with semiamplitudes of  $\sim 28 \text{ km s}^{-1}$  in the primary and  $\sim 188 \text{ km s}^{-1}$  in the secondary. They infer a secondary mass of  $\sim 50 M_J$  (where  $M_J$  is the mass of Jupiter) — clearly above the canonical

“planet” mass but less massive than the hydrogen-burning mass limit (Burrows et al. 2001). Provocative evidence of very close low-mass companions to an evolved star was reported by Charpinet et al. (2011), who claim to find two sub-Earth-radius planets around the subdwarf star KIC 05807616 that are both orbiting inside  $1.7 R_{\odot}$  (where  $R_{\odot}$  is the radius of the Sun). If these planet candidates are actually planets, this discovery shows that exotic and theoretically unanticipated post-main-sequence planetary systems are possible. Hogan et al. (2009) report a nondetection of warm companions around 23 nearby white dwarfs and suggest that  $\lesssim 5\%$  of white dwarfs have companions with effective temperatures greater than 500 K between 60 AU and 200 AU in projected separation. Several years ago, the pulsating white dwarf GD-66 was found to exhibit timing variations in pulsations that seemed to be consistent with light-travel-time delays caused by orbital motion around the common center of mass with a companion on a  $\sim 4.5$ -year orbit (Mullally et al. 2007, 2008, 2009). However, recent high-precision astrometric observations of this system, an analysis of *Spitzer* observations of the system, and follow-up observations by the team of original discovery, have complicated the planetary hypothesis for this system (Farihi et al. 2012; J. Hermes 2012, private communication). Finally, Johnson et al. (2011) has found a number of giant planets around slightly evolved, subgiant stars. A number of lines of evidence, then, point to the existence of planets around evolved stars; even if some of the candidate systems eventually turn out not actually to be planets, it still seems clear that several tens of percent of white dwarfs have planets around them (Zuckerman et al. 2010).

It is worthwhile to recognize that the existence of planets orbiting post-main-sequence stars should be no surprise, given that the very first planets discovered beyond our solar system were found around the millisecond pulsar PSR1257+12 (Wolszczan & Frail 1992). Whether planets around stellar remnants formed after the death of the primary, or were present during the main-sequence phase and survived stellar evolution, remains an open question. It seems likely that both processes occur (Hansen et al. 2009; Tutukov & Fedorova 2012); although there might be more reason to believe that planets could form around a pulsar than around a white dwarf, there is no consensus as yet.

Many jovian companions to main-sequence stars will become highly irradiated as their primaries evolve off the main sequence, thereby turning them into “hot Jupiters,” of sorts — or red-giant hot Jupiters, as described by Spiegel & Madhusudhan (2012). The atmospheres of these companions might become transiently polluted by accretion both of the evolved star’s wind and of dust and planetesimals (Spiegel & Madhusudhan 2012; Dong et al. 2010). Some of these planetary companions, or somewhat higher mass companions, will eventually be swallowed by their stars, which might contribute to the formation and morphology of planetary nebulae (Nordhaus & Blackman 2006; Nordhaus et al. 2007) and to the formation of highly magnetic white dwarfs (Tout et al. 2008; Nordhaus et al. 2011), although there are other formation models in the literature as well (García-Berro et al. 2012).

The remainder of this document is structured as follows: In §2, I show that the increased moment of inertia of an evolved star can cause a companion that had been slowly moving outward due to tidal torques to instead rapidly plunge into the primary. In §3, I argue that simple binary

evolution leads to two populations of companions to white dwarfs, with a large gap in period (or orbital radius) between them. In §4, I comment briefly on the potential sub-Earth-sized planets around KOI 55 found by Charpinet et al. (2011). Finally, in §5, I summarize and conclude.

## 2. Evolution of Tidal Torques

Consider two bodies of masses  $M_1$  and  $M_2$  each in a circular orbit around the common center of mass. The angular momentum of the orbit is  $L_{\text{orb}} = \mu a^2 n$ , where  $\mu \equiv M_1 M_2 / (M_1 + M_2)$  is the reduced mass,  $a$  is the orbital semimajor axis, and  $n = 2\pi/P$  is the orbital mean motion (where  $P$  is the orbital period). If the moments of inertia of the two bodies are  $I_1$  and  $I_2$  and their angular rotation rates are  $\Omega_1$  and  $\Omega_2$ , then their respective spin angular momenta are  $L_1 = I_1 \Omega_1$  and  $L_2 = I_2 \Omega_2$ . The total angular momentum of the system is

$$L = L_1 + L_2 + L_{\text{orb}} = I_1 \Omega_1 + I_2 \Omega_2 + \mu a^2 n. \quad (1)$$

There are no net tidal torques (and therefore the system is tidally locked) if each member of the binary is spinning at the orbital mean motion. In other words, tidal equilibrium fixed points are obtained when the system’s angular momentum is equal to

$$L = I_1 n_{\text{eq}} + I_2 n_{\text{eq}} + \mu a_{\text{eq}}^2 n_{\text{eq}}, \quad (2)$$

where  $n_{\text{eq}}$  and  $a_{\text{eq}}$  are the equilibrium mean motion and orbital radius, respectively, for the given system angular momentum  $L$ . Since  $n = \sqrt{G(M_1 + M_2)/a^3}$ , this may be rewritten as (dropping the “eq” subscripts),

$$L = a^{-3/2} \sqrt{G(M_1 + M_2)} \left( I_1 + I_2 + \frac{M_1 M_2}{M_1 + M_2} a^2 \right). \quad (3)$$

Equation (3) has two striking features. First,  $L$  approaches infinity as  $a$  approaches both zero and infinity. Second (and as a consequence), for any (sufficiently great) total angular momentum of the system, there are two orbital radii at which the binary can achieve tidal equilibrium. This was pointed out by Darwin (1879, 1880), who noted that the Earth-Moon system could be in tidal equilibrium in a  $\sim 5$ -hour orbit, in addition to the more commonly recognized  $\sim 50$ -day orbit that we are slowly approaching. A perturbative stability analysis shows that the inner fixed point is unstable — a “repeller,” and the outer one is stable — an “attractor” (Hut 1980). A system that has an orbital radius between the two fixed points approaches the outer fixed point, as the Earth-Moon system is doing. As pointed out by Levrard et al. (2009) and others, a system that is inside the inner fixed point is drawn inexorably by tidal torques toward a merger (which, in the case of planet-star mergers can produce spectacular optical, ultraviolet, and X-ray signatures, as described by Metzger et al. 2012).

For a system with companion that has a small radius and small mass relative to the primary, equation (3) may be simplified as follows:

$$L \approx a^{-3/2} \sqrt{GM_*} (I_* + M_c a^2) \quad (4)$$

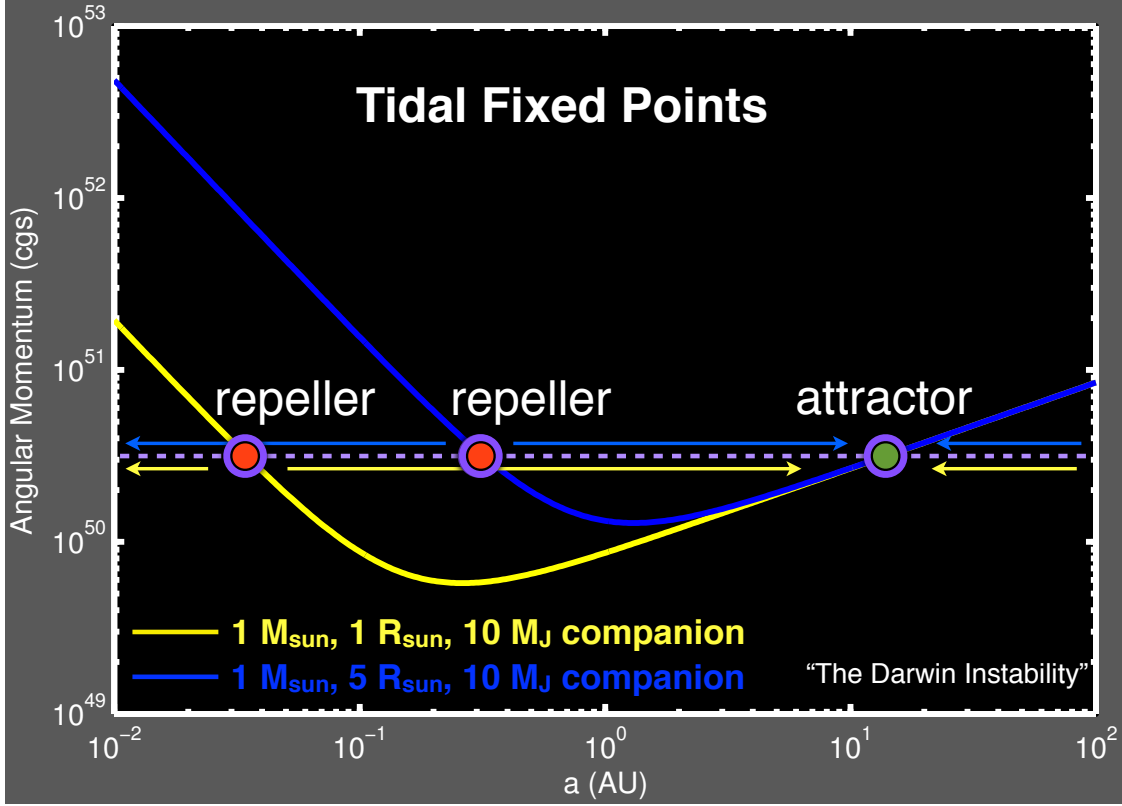


Fig. 1.— Tidal fixed points. The curves represent orbital radius/angular momentum pairs for a  $1-M_{\odot}/10-M_{\text{Jup}}$  binary at which tidal fixed points are possible (i.e., where it is possible for both binary components to be spinning with angular frequencies equal to the orbital mean motion). The yellow curve represents the locus of fixed points assuming a main-sequence solar-mass star as the host; the blue curve represents the locus of fixed points assuming a post-main-sequence ( $5-R_{\odot}$ ) star as the host. The horizontal purple dashed line indicated a particular total system angular momentum; note that for this (and for many) angular momenta there are two fixed points, a close-in one and a more distant one. The inner one is a repeller and the outer one is an attractor.

$$= a^{-3/2} \sqrt{GM_*} (\alpha_* M_* R_*^2 + M_c a^2), \quad (5)$$

where the subscripts  $*$  and  $c$  refer to the primary star and the companion, respectively, and in equation (5) we have rewritten the primary’s moment of inertia using  $I_* \equiv \alpha_* M_* R_*^2$  for some value  $\alpha_*$ . Although the  $\alpha_*$  value of an evolving star is almost surely a function of  $R_*$  (or alternatively of time), if we take  $\alpha_*$  to be constant then we may examine the approximate behavior of the  $L$ — $a$  relation in two limiting cases. For  $M_c a^2 \ll \alpha_* M_* R_*^2$  (i.e., near the close-in fixed point),

$$a_{\text{close}}[L] \sim G^{1/3} \alpha_*^{2/3} L^{-2/3} M_* R_*^{4/3} \propto R_*^{4/3}; \quad (6)$$

whereas for  $M_c a^2 \gg \alpha_* M_* R_*^2$  (near the more distant fixed point),

$$a_{\text{distant}}[L] \sim G^{-1} L^2 M_*^{-1} M_c^{-2}, \quad (7)$$

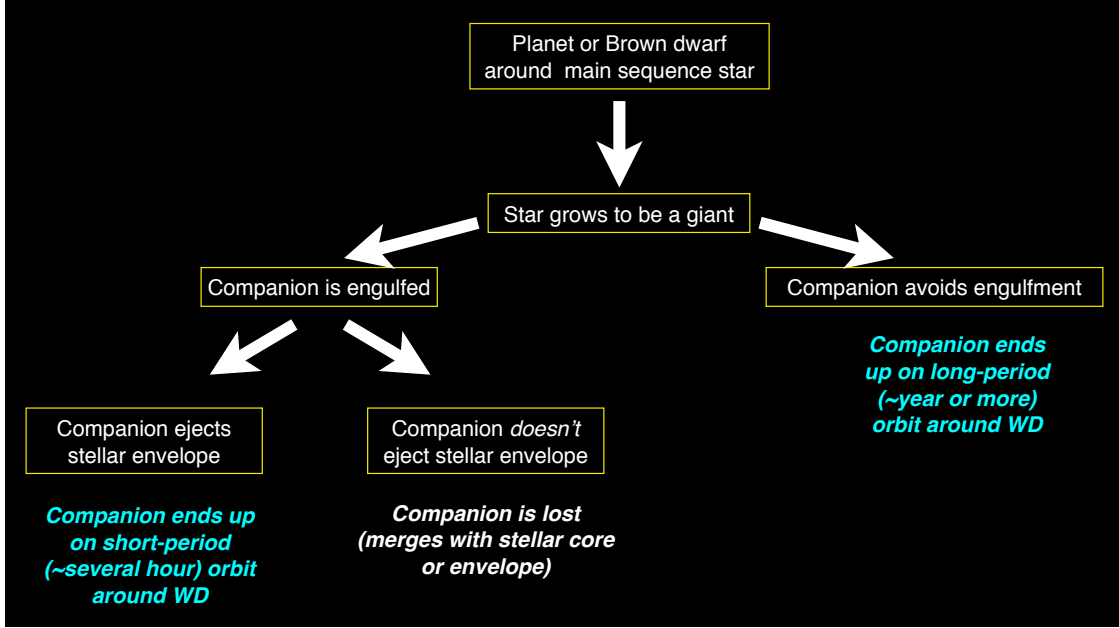
and is nearly independent of  $R_*$ .

Figure 1 illustrates what happens when one member of the binary (the more massive member) rapidly changes its moment of inertia (such as when a star ascends the red giant branch (RGB), increasing its moment of inertia at constant mass and therefore constant binary angular momentum). The yellow and blue curves represent the set of tidal fixed points for, respectively, a main-sequence solar-type star, and a post-main-sequence solar-type star (here, a  $5-R_\odot$  primary with a  $10-M_J$  companion, where  $M_J$  is Jupiter’s mass). The inner part of the yellow curve moves outward in orbital radius, in accordance with equation (4), while the outer part remains essentially fixed, as per equation (5).

As the repeller point approaches a planet, what happens? It is tempting to think that the repeller would push the planet outward, in the direction of the attractor point. In truth, though, the outcome depends on the relative timescales of stellar evolution and tidal evolution. The rate of tidal evolution depends on  $(R_*/a)^6$ , where  $R_*$  is the primary’s radius and  $a$  the companion’s orbital radius. For reasonable values of the primary’s tidal quality factor  $Q'_*$ , a companion at an orbital radius greater than a few tenths of an AU has a tidal evolution timescale that is long compared with the primary’s evolution timescale. As a result, the primary’s evolution and increasing moment of inertia causes the repeller point to move past the companion. In this manner, a system that had been gently evolving toward the outer (stable) tidal fixed point will suddenly find itself on the inside of the unstable fixed point, thereupon suffering the “Darwin Instability” and being tidally dragged toward a merger with the star. This process accelerates as the primary’s ascent of the RGB (and increasing radius) eventually leads to fast tidal evolution.

This process is guaranteed to happen, irrespective of the nature of tidal dissipation. Details of the physics of tidal dissipation affect the strength of the tidal torque and therefore the *rate* at which tidal interactions influence the orbit, but the fact that post-main-sequence evolution can turn a stable evolving system into an unstable plunging one is a simple consequence of conservation of angular momentum.

### 3. Two Populations



The above flowchart summarizes the possible evolutionary paths that a binary system can follow. In short, there are three types of outcomes: a companion can avoid ever being engulfed by the primary; or a companion can merge with the evolving star and then either survive the common envelope (CE) phase or be destroyed.

What properties of a binary system affect the eventual outcome? The primary’s mass loss tends to make the orbit expand in proportion to  $M_*^{\text{ZAMS}}/M_*[t]$ , where  $M_*^{\text{ZAMS}}$  is the zero-age main-sequence mass of the star, and  $M_*[t]$  is its mass at time  $t$ , while tidal torques will generally tend to make the orbit shrink (so long as the orbital timescale is shorter than the evolved star’s rotational period). Other effects that might be thought to influence a binary companion’s orbit, such as drag from moving through the enhanced stellar wind, are negligible for planetary-sized (or more massive) companions (Villaver & Livio 2009).

Those companions that avoid engulfment must be at least an AU away from the star (probably actually several-to-20 AU), and those that survive a CE end up very close ( $\lesssim 0.01$  AU for planetary and brown-dwarf-mass companions). As a result, there ought to be a large gap in orbital radius (or period) in the distribution of binary companions to white dwarfs. The approximate boundaries of the gap may be estimated as described below:

If a companion is to avoid ever being swallowed, it must be far enough from the primary that tidal torques cannot sap it of enough orbital energy to make it plunge. Carlberg et al. (2009), Villaver & Livio (2009), and Nordhaus et al. (2010) investigated the set of initial planetary orbits that lead to mergers and found that if a several-Jupiter-mass object begins at least a few times the maximum stellar radius achieved during the RGB or asymptotic giant branch (AGB) phase, it will avoid being tidally engulfed (where “a few” means  $\sim 3$ – $6$ , for a fiducial stellar tides prescription, as calibrated by Verbunt & Phinney 1995). Since the maximum stellar radius for a  $1$ – $3 M_\odot$  star

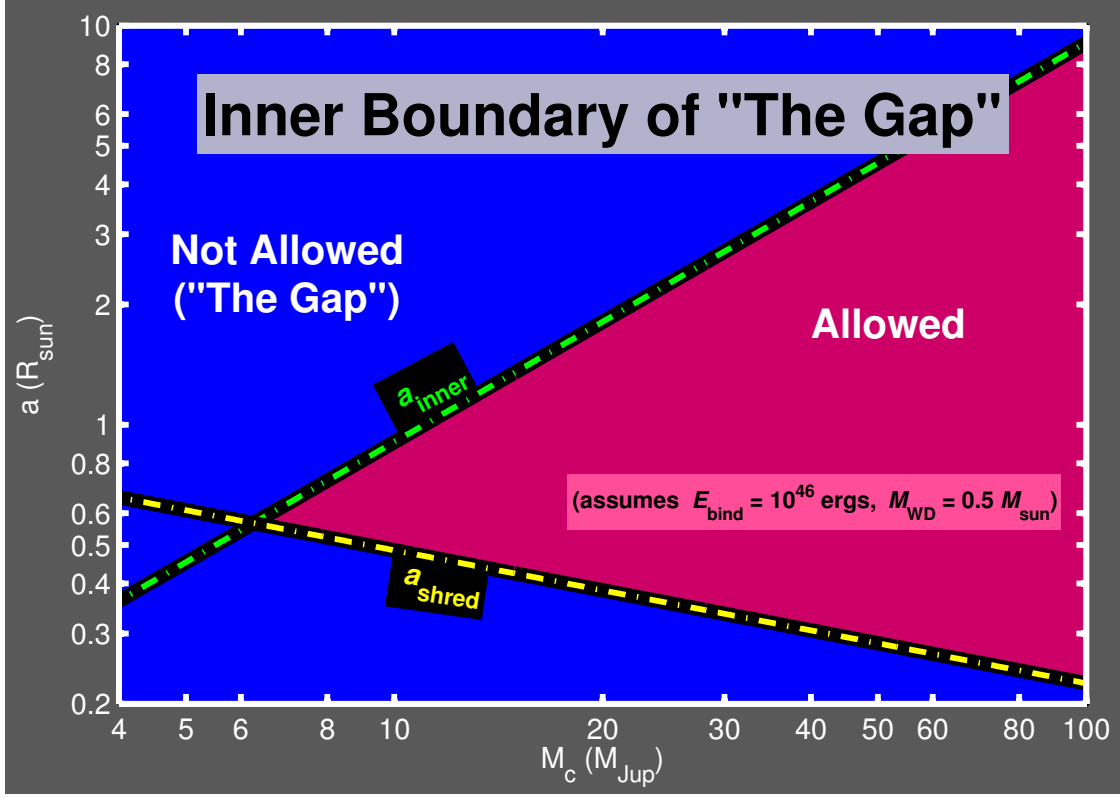


Fig. 2.— Interior to the period/orbital-radius gap for binary companions to white dwarfs. The blue region is inaccessible to binary companions. The magenta region, consisting of companions more massive than  $\sim 6 M_J$  and inside a few  $R_\odot$  in orbital radius, is energetically allowed for binary companions and yet is far enough from the white dwarf that a  $1-R_J$  companion will not be tidally shredded. This figure assumes an envelope binding energy of  $10^{46}$  ergs and a white dwarf mass of  $0.5 M_\odot$ . Higher envelope binding energies result in companions that are closer to the white dwarf (and in a greater minimum companion mass). The green dashed-dotted line indicates  $a_{\text{inner}}$  (equation 9), below which a companion can dissipate enough orbital energy to unbind a  $10^{46}$ -erg envelope, and the yellow dashed-dotted line indicates  $a_{\text{shred}}$  (equation 12), above which a  $1-R_J$  companion avoids tidal disruption.

is in the range of one to several AU, this suggests that the outer boundary of the gap ranges from a few to  $\sim 20$  AU, depending on the progenitor star’s main-sequence mass.

If a companion is to survive a common envelope (Soker et al. 1984), it must be massive enough that the energy it injects into the stellar envelope is sufficient to unbind the star (before the companion is tidally disrupted). If the mass of the star’s degenerate core’s mass is  $M_{\text{WD}}$ , then the orbital energy lost as the companion sinks through the CE to an orbital radius of  $a$  is  $\Delta E_{\text{orb}} \sim -GM_{\text{WD}}M_c/2a$ . Therefore, if the stellar envelope’s binding energy is  $E_{\text{bind}}$ , the radius of the inner edge of the gap may be approximated as

$$a_{\text{inner}} \sim \frac{GM_{\text{WD}}M_c}{2E_{\text{bind}}} \quad (8)$$

$$\sim 0.9R_{\odot} \left( \frac{M_c}{10M_{\text{Jup}}} \right) \left( \frac{M_{\text{WD}}}{0.5M_{\odot}} \right) \left( \frac{E_{\text{bind}}}{10^{46} \text{ erg}} \right)^{-1}. \quad (9)$$

Equation (9) assumes that all the lost orbital energy goes into unbinding the stellar envelope. If only a fraction  $\alpha_{\text{CE}}$  of this energy actually participates in unbinding the envelope (for further details on CE alpha, see Soker 2012), then the actual maximum orbital radius at the inner edge of the gap is reduced by a factor of  $\alpha_{\text{CE}}$ . Similarly, if, for some system,  $\alpha_{\text{CE}}$  were greater than unity, the maximum orbital radius at the inner edge of the gap could be greater than that indicated in equation (9), but I am aware of no suggestion that  $\alpha_{\text{CE}}$  could be dramatically larger than 1 for a companion that survives the CE.<sup>1</sup> The orbital period at the inner edge of the gap is

$$P_{\text{inner}} \sim \left( \frac{4\pi^2 a_{\text{inner}}^3}{GM_{\text{WD}}} \right)^{1/2} \\ \sim 3.4 \text{ hrs} \times \left( \frac{M_c}{10M_{\text{Jup}}} \right)^{3/2} \left( \frac{M_{\text{WD}}}{0.5M_{\odot}} \right) \left( \frac{E_{\text{bind}}}{10^{46} \text{ erg}} \right)^{-3/2}. \quad (10)$$

Figure 2 illustrates the region of parameter space that is allowed at the inner edge of the gap, as a function of companion mass and orbital radius. Not only is there a maximum orbital radius for the inner edge of the gap, but there is a minimum radius too, defined by the tidal shredding radius

$$a_{\text{shred}} \sim R_c \left( \frac{2M_{\text{WD}}}{M_c} \right)^{1/3} \quad (11)$$

$$\sim 0.5R_{\odot} \times \left( \frac{R_c}{R_J} \right) \left( \frac{M_{\text{WD}}}{0.5M_{\odot}} \right)^{1/3} \left( \frac{M_c}{10M_J} \right)^{-1/3}, \quad (12)$$

where  $R_c$  is the companion’s radius and  $R_J$  is Jupiter’s radius. The maximum and minimum orbital radii ( $a_{\text{inner}}$  and  $a_{\text{shred}}$ ) are equal for a companion mass of

$$M_c^{\text{min}} \sim \left( \frac{16R_c^3 E_{\text{bind}}^3}{G^3 M_{\text{WD}}^2} \right)^{1/4} \quad (13)$$

$$\sim 6M_J \times \left( \frac{R_c}{R_J} \right)^{3/4} \left( \frac{E_{\text{bind}}}{10^{46} \text{ erg}} \right)^{3/4} \left( \frac{M_{\text{WD}}}{M_{\odot}} \right)^{-1/2}. \quad (14)$$

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<sup>1</sup>If a hydrogen-rich companion, such as a Jupiter, falls deep enough into an AGB star’s interior, this could deposit fresh hydrogen in a helium-burning layer, which might help to donate significantly more energy to the envelope than simply the lost orbital energy, but at the cost of the companion’s survival, as noted by Nordhaus et al. (2011).



Companions less massive than this ought to be shredded and incorporated into the stellar envelope before they lose enough orbital energy to unbind the stellar envelope, and consequently it would be a mystery if they were to survive the CE phase (although, were low-mass planets to form after the formation of the white dwarf, they might still be found inside the gap region).

#### 4. sdB Planets

Charpinet et al. (2011) recently announced the discovery of a puzzling object in the Kepler data. KOI-55 is a subdwarf-B (sdB) star of radius  $\sim 0.2 R_{\odot}$  whose lightcurve shows regular variations with two distinct periods ( $\sim 8.2$  hours and  $\sim 5.8$ ) in a 10:7 ratio. Charpinet et al. argue that these variations are inconsistent with any intrinsic pulsational periods in a star of this type and that the most plausible explanation is the presence of two sub-Earth-radius (nontransiting) planets whose daysides periodically come into view.

It is difficult to see how these putative objects could have arrived at their orbits through ordinary CE evolution. For stability reasons, their masses must be significantly less than Jupiter’s, as noted by Charpinet et al. and as can easily be verified with an N-body integrator such as, e.g., **REBOUND** (Rein & Liu 2012). For instance, if they are as massive as Jupiter, one of them would probably be ejected from the system in less than a year. Since they must be significantly less massive than Jupiter, the objects that are currently in orbit around the star could not have deposited enough orbital energy in the stellar envelope to unbind it. The same stability argument suggests that it would require extreme fine tuning for two massive ( $\gtrsim 10 M_J$ ) bodies to move through a common envelope together, in resonance, and evaporate down to  $\sim$ Earth-sized cores at their present locations. Bear & Soker (2012) argue that these objects might be the result of a single massive companion that was tidally shredded during a CE phase. In this scenario, the core was shredded, too, and two chunks of the core were flung from the shredding radius to their current orbits. Another conceivable scenario, if these lightcurve variations correspond to actual companions, is that both objects migrated to their current locations after the sdB star formed from a previous CE companion that was destroyed during the CE phase.<sup>2</sup> This scenario would require an outer perturber, and would not be a result of simple binary CE evolution. The planet hypothesis for this puzzling lightcurve might be true, but it is not obvious how the planets could have reached their present orbits.

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<sup>2</sup>This migration process might have occurred via, e.g., Kozai interactions followed by tidal circularization (Kozai 1962; Fabrycky & Tremaine 2007; Katz et al. 2011; Socrates et al. 2012; Naoz et al. 2012; Shappee & Thompson 2012).

## 5. Conclusion

Post-main-sequence stellar evolution results in dramatic changes in the stellar radius and, therefore, in the orbits of companions to the stars. Companions that are too close can either be directly swallowed by the expanding star or tidally dragged into merging with the star. The ensuing common envelope poses severe risks to the survival of the companion; massive enough companions can eventually transfer enough orbital energy to unbind the star, resulting in a tight post-CE orbit, while less massive companions continue spiraling inwards until they are tidally shredded and merge with their host stars. Companions that are on distant enough orbits to avoid ever merging with their host stars move outward due to mass loss from the primary.

Nearly all objects that are massive enough to survive a CE have masses in excess of the deuterium-burning limit that is sometimes used to delineate between planets and brown dwarfs (Spiegel et al. 2010). For an object that is less than  $\sim 6$  times the mass of Jupiter to end up inside 1 AU from a WD requires something more complex than simple binary/CE evolution. In particular, forming the potentially habitable circum-white-dwarf terrestrial objects considered by Agol (2011) and Fossati et al. (2012) might require exotic circumstances involving multiple bodies, and the evolutionary paths to produce such worlds might be problematic for their subsequent habitability.

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